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In the computation of square roots using trigonometric exact

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ABSTRACT
The field of mathematics is flooded with a slew of iterative methods for finding square roots, which converge after some specific number of steps. In this paper, we present a direct approach in finding the square root $\sqrt{S^2}$. We consider the exact trigonometric function $\tan x$, for finding square roots considering the right-angle triangle with opposite s, adjacent 1. Results from this approach indicate that we get an exact non iterative value for the value of a square root.

I. INTRODUCTION

Square roots have a wide implementationin platforms found in digitalsignal processing and science in general such as microcontrollers and other power control electronics, computer graphics, vision apparatus, video games, multimedia, and in solving for linear equations in mathematics [1, 2, 3, 4, 5,6].

The square root methods that compute for xrange [7, 8, 9, 10]from iterative methods [16, 19], polynomial methods [4, 16, 17, 20], table-based methods [7, 11, 12, 13], digit recurrence methods [7,9, 14] and rational methods. The iterative methods like Newton Rhapson (Babylonian method) and Goldschmidt have quadratic convergence and can be used in platforms that do not require floating point calculations (calculators and some microprocessors) [2]. These are fast converging methods but they require a good approximation of the initial guess function. [2, 15, 16, 17, 18]. The initial approximation of**s**as a starting functionoften takes**n** iterative steps [2]. Our exact method $s = \tan x$surpasses the need to come up with an initial approximation value and it takes only one step thus it avoids the need to take **n** iterative steps.

Table-based methods are fast but require more computer memory [2]. Polynomial methods are fast but require more computer memory for the storage of the polynomial coefficients, they tend also to show low accuracy in square root calculations as there involve a long string of infinite terms [2]. Floating point square rooting methods have increased computational cost than addition and subtraction [19]. The following work will introduce trigonometric and exponential functions for the exact computation of square roots. Since the method presented in this work of calculating square roots involve$\tan x$, perturbation series of $\sin x$, and exponentials thus only multiplication and subtraction operations are utilized.These operations can be implemented on platforms that don't require floating point arithmetic i.e. personal calculators and some microprocessors and field programmable gate arrays, with some inherent speed and accuracy despite the need for increased storage of the perturbation coefficients.

Dianov et al. [1] did a review of fast square rooting computation methods for fixed point microcontroller-based control systems of power electronics. They also came up with a more robust fixed point arithmetic algorithm that can be implemented in any hardware or software that is not based on the efficiency of the platforms in use. Their method is quasi-non-iterative in the case that it only involves one Newton iterative step to improve accuracy. The method requires initial approximation of the root x using a secant line given by;

$$Y(x) = \frac{y_1 - y_2}{x_2 - x_1}x + \frac{y_2x_1 - y_1x_2}{x_1 - x_2}x \quad (1)$$

The square root is found as:

$$x = \frac{y_1x_2 - y_2x_1}{y_1 - y_2} \quad (2)$$

Look up table-based methods that are widely used in science and engineering for function approximation. They are relatively fast but require large storage facilities. Their inherent error in precision can be reduced by linear interpolation between table points. They can be used to

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approximate trigonometric functions like sines and cosines.Polynomials are also used in square rooting calculations but due to the rapid change of derivatives of square root near zero, they give a large error in Taylor expansion and require a very small convergence radius.

Rhapson MethodIs a method of finding an iterative approximation to the real value of a function using an initial guess root x_1. The general formula for the algorithm is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}. \quad (3)$$

The process is iterated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \quad (4)$$

The convergence of the Newton-Rhapson method is demonstrated using the Taylor series and its is shown to converge quadratically. That is, from equation (4), it follows that,

$$f(x) = 0 = f'(x_n) + f'(x_n)(x - x_{n+1}) + \frac{f''(\alpha)(x - x_n)}{2}. \quad (5)$$

Note $x_n < \alpha < x_{n+1}$ and $f''(x_n) = 2x$ and $f''(\alpha) = 2$, looking at the second term Is always 1 so the high order terms vanishes in the Taylor series showing that the Newton-Rhapson method converges quadratically.

II. PROPOSED METHOD

1.1 Exact tan method for solving for square root

The method proposed in the current study utilises trigonometric functions. It gives a direct analytic solution without needing to perform iterative steps. It is emerging as a function that employs trigonometric functions and yield an accurate value of the root of a number. The level of accuracy is on direct precision hence approximation errors become automatically curbed.

Using the tan trigonometric function, it is shown that all square roots have conjugate angles that can be operated on by the function tan to yield a relevant square root. Consider a schematic of a typical right angled triangle I figure 1, calibrated as Sbeing the opposite side to the angle θ, while 1 unit represent the measurement adjacent side. The trigonometric ratio follows as;

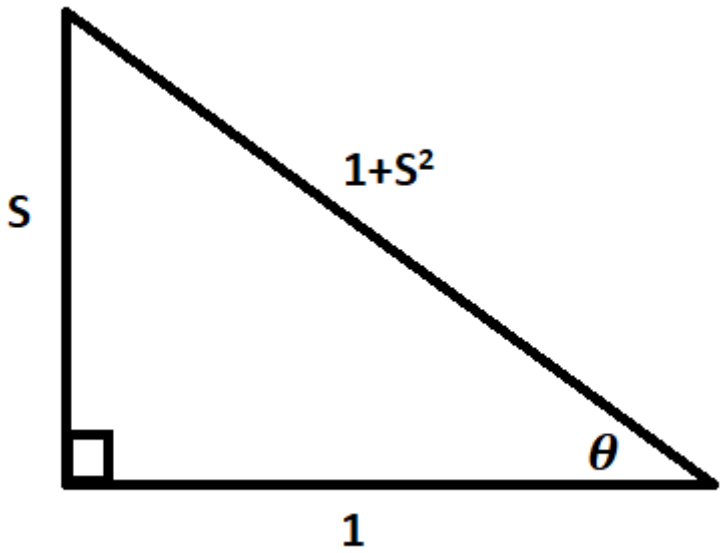

Figure 1. Schematic of the right-angled triangle

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The trigonometric ratio follows as;

$$\sin^2\theta = \frac{S^2}{1+S^2}. \quad (6)$$

And

$$\cos^2\theta = \frac{1}{1+S^2}. \quad (7)$$

Thus

$$S^2 = \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}. \quad (8)$$

Also we have the identity $\cos^2\theta = 1 + 2\sin^2\theta$. This implies that,

$$x = \frac{1}{2}\cos^{-1}(1 - 2\sin^2\theta). \quad (9)$$

Hence

$$S = \tan\left(\frac{1}{2}\cos^{-1}(1 - 2\sin^2\theta)\right). \quad (10)$$

This therefore gives the formulation of the square root as;

$$S = \tan\left(\frac{1}{2}\cos^{-1}\left(\frac{1-s^2}{1+s^2}\right)\right). \quad (11)$$

III. APPLICATION OF THE METHOD

Consider the following using the formulation given by equation (11)

Number	Calculator Square Root	Equation (11)
16	4	4
25	25	24
55	7.4162	7.4162

Table1. Application of the square root formula.

Table 1. shows a comparison of the square root formula with comparison with direct calculation. The formula gives exact same results.

IV. CONCLUSION

The current work gives an accurate method to find the square root of a number. It uses trigonometric functions. This method is accurate and direct, and it does not require any iterative steps. This is a major contribution in computational mathematics as it can be coded as an alternative to calculation of roots of numbers.

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